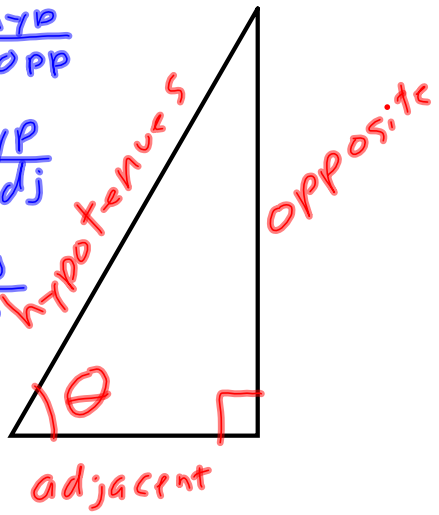
4.3 Right Triangle Trigonometry

$\sin\theta = \frac{opp}{hyp}$ $\quad \csc\theta = \frac{hyp}{opp}$

$\cos\theta = \frac{adj}{hyp}$ $\quad \sec\theta = \frac{hyp}{adj}$

$\tan\theta = \frac{opp}{adj}$ $\quad \cot\theta = \frac{adj}{opp}$



$\sin\theta = \frac{4}{5}$ $\quad \csc\theta = 5/4$

$\cos\theta = 3/5$ $\quad \sec\theta = 5/3$

$\tan\theta = 4/3$ $\quad \cot\theta = 3/4$

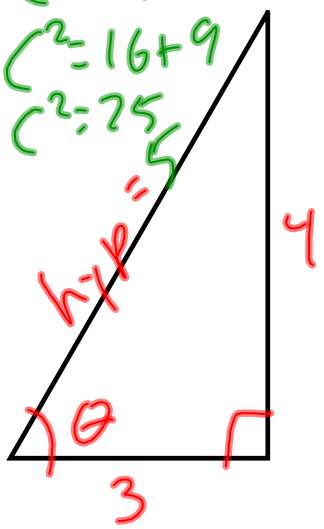
$c^2 = a^2 + b^2$

$c^2 = 4^2 + 3^2$

$c^2 = 16 + 9$

$c^2 = 25$

$c = 5$



Given $\sin\theta = \frac{3}{8}$

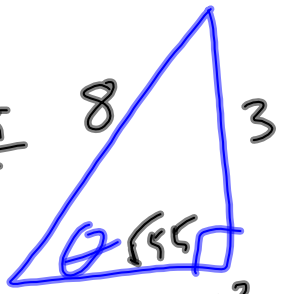
Sketch the triangle and find other five trig. functions.

$\sin\theta = \frac{3}{8}$ $\quad \csc\theta = \frac{8}{3}$

$\cos\theta = \frac{\sqrt{55}}{8}$ $\quad \sec\theta = \frac{8}{\sqrt{55}} = \frac{8\sqrt{55}}{55}$

$\tan\theta = \frac{3\sqrt{55}}{55}$ $\quad \cot\theta = \frac{\sqrt{55}}{3}$

$\frac{3}{\sqrt{55}} \cdot \frac{\sqrt{55}}{\sqrt{55}} = \frac{3\sqrt{55}}{55}$



$c^2 = a^2 + b^2$

$8^2 = a^2 + 3^2$

$64 - 9 = a^2$

$a = \sqrt{55}$

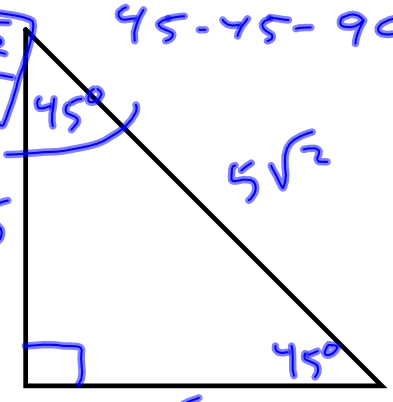
Special Right Triangles

45-45-90

$\sin 45^\circ = \frac{5}{5\sqrt{2}} = \frac{1}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \boxed{\frac{\sqrt{2}}{2}}$

$\cos 45^\circ = \frac{5}{5\sqrt{2}} = \frac{\sqrt{2}}{2}$

$\tan 45^\circ = \frac{5}{5} = 1$



$c^2 = 5^2 + 5^2$

$c^2 = 50$

$c = \sqrt{50} = \sqrt{25 \cdot 2}$

$= 5\sqrt{2}$

$c^2 = a^2 + b^2$

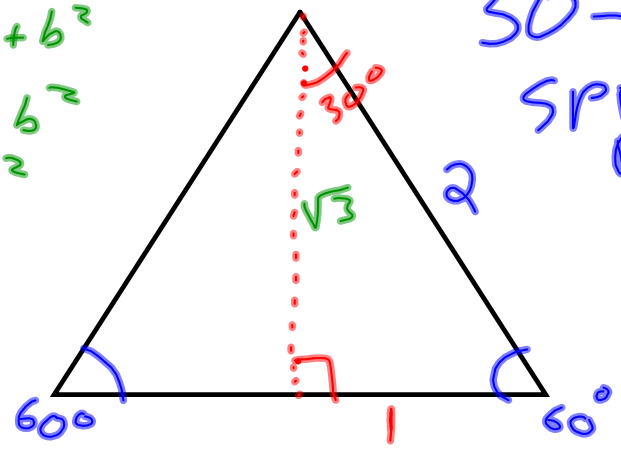
$2^2 = 1^2 + b^2$

$4 = 1 + b^2$

$3 = b^2$

$b = \sqrt{3}$

30-60-90 special Right Triangle



$\tan 30° = \frac{1}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{\sqrt{3}}{3}$

$\sin 30° = \frac{1}{2}$ $\sin 60° = \frac{\sqrt{3}}{2}$

$\cos 30° = \frac{\sqrt{3}}{2}$ $\cos 60° = \frac{1}{2}$

$\tan 30° = \frac{\sqrt{3}}{3}$ $\tan 60° = \sqrt{3}$

Cofunctions: Complimentary Angles have same trig. ratio

$\sin(90° - \theta) = \cos\theta$

$\cos(90° - \theta) = \sin\theta$

$\tan(90° - \theta) = \cot\theta$

$\sec(90° - \theta) = \csc\theta$

Trig. Identities

ex $\sin(90° - 35°) = \cos 55°$

Finding angle given trig. ratio

$\sin\theta = \frac{3}{8} \rightarrow \sin^{-1}\frac{3}{8} = \theta$

↑ sin inverse

Calc:

2nd [Sin] 3 / 8 [Enter] $= 22.02^\circ$

note: check [MODE] be in degrees

$\tan\theta = \frac{7}{2}$ $\tan^{-1} 7/2 = \theta$

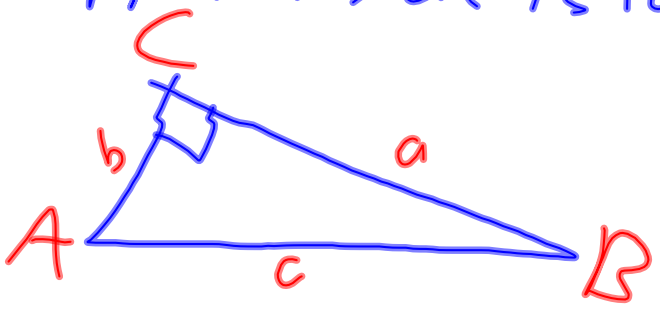
$\theta = 74.05^\circ$

Labeling a triangle

↑ right

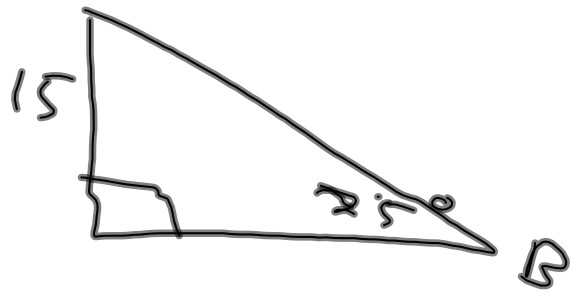
Angles are capital letters

opposite side is lowercase version



Right angle labeled with Capital C

ex Triangle ABC has angle $b = 25^\circ$ side $b = 15$



4.3 Continued

Fundamental Trig. Identities

Reciprocal Identities

$\sin\theta = \frac{1}{\csc\theta}$ $\quad \csc\theta = \frac{1}{\sin\theta}$

$\cos\theta = \frac{1}{\sec\theta}$ $\quad \sec\theta = \frac{1}{\cos\theta}$

$\tan\theta = \frac{1}{\cot\theta}$ $\quad \cot\theta = \frac{1}{\tan\theta}$

Quotient Identities

$\tan\theta = \frac{\sin\theta}{\cos\theta}$ $\quad \cot\theta = \frac{\cos\theta}{\sin\theta}$

ex $\tan 35^\circ = \frac{\sin 35^\circ}{\cos 35^\circ}$

Pythagorean Identities

$\sin^2\theta + \cos^2\theta = 1$

$1 + \tan^2\theta = \sec^2\theta$

$1 + \cot^2\theta = \csc^2\theta$

ex Let θ be an angle (acute) such that $\sin\theta = .6$

find $\cos\theta$ & $\tan\theta$

since $\sin^2\theta + \cos^2\theta = 1$

$(.6)^2 + \cos^2\theta = 1$

$.36 + \cos^2\theta = 1$

$\cos^2\theta = .64$

$\cos\theta = .8$

since $\tan\theta = \frac{\sin\theta}{\cos\theta}$

$\tan\theta = \frac{.6}{.8}$

$\tan\theta = .75$

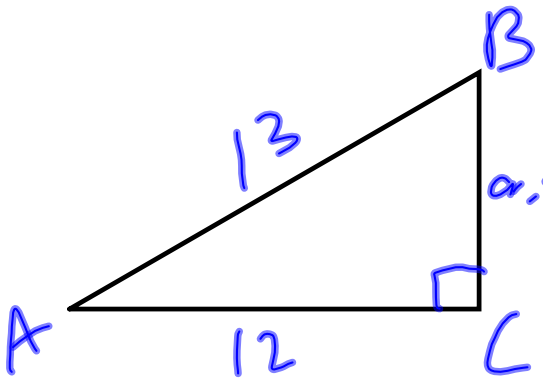
Can use identities to turn one side of an equation into the other side.

ex show that

$\cos\theta \sec\theta = 1$

$\cos\theta \frac{1}{\cos\theta} = 1$

$\frac{\cos\theta}{\cos\theta} = 1$



$\sin A = \frac{5}{13}$ $m\angle A = 22.6°$

$\cos A = \frac{B}{13}$ $m\angle B = 67.4°$

$\tan A = 5/12$ $m\angle C = 90°$

$90 - 22.6 =$

$13^2 = 12^2 + a^2$

$13^2 - 12^2 = a^2$

$25 = a^2$

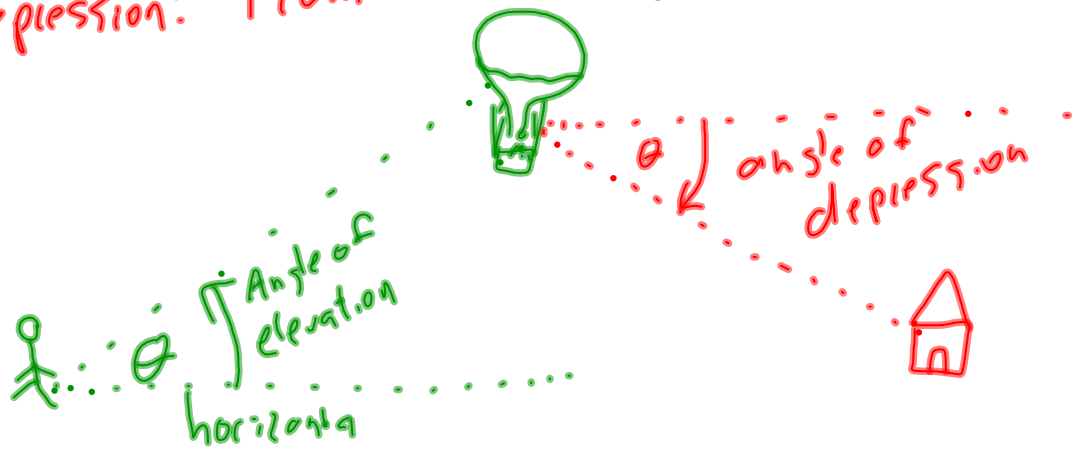
$5 = x$

4.3 Conclusion

Angle of Elevation / Angle of Depression

Elevation: From horizontal up to object

Depression: From horizontal down to object



Ex A surveyor is standing 50 feet from the base of a large tree. The angle of elevation to the tree top is 71.5°. How tall is the tree.

$\tan 71.5 = \frac{x}{50}$

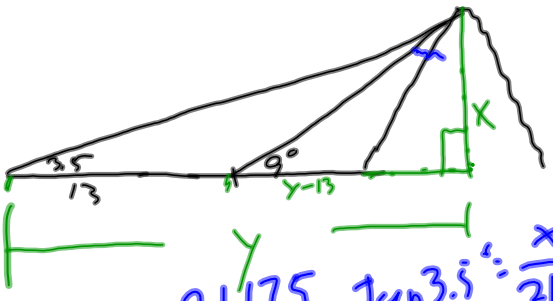
$x = \tan 71.5 \cdot 50$

$x = 149.4'$

Height of a Mountain.

In traveling across flat land you see a mountain in front of you. It's angle of elevation to the peak is 3.5°. After driving 13 miles closer the angle of elevation is 9°. Approximate the height of the mountain

$$\tan 3.5^\circ = \frac{x}{y} \qquad \tan 3.5y = x \qquad .061y = x$$



$$\tan 9^\circ = \frac{.061y}{y-13}$$

$$21.175 \quad \tan 3.5^\circ = \frac{x}{21.175} \qquad .158 = \frac{.061y}{y-13}$$

$$\tan 9^\circ = \frac{x}{y-13}$$