

11.2 Simplify Radical Expressions



A radical expression is in simplest form when:

- No perfect square factors in radicand.
 ~~$\sqrt{8}$~~ $\sqrt{4 \cdot 2} = \sqrt{4} \cdot \sqrt{2} = 2\sqrt{2}$
- No fractions in radicand.
 ~~$\sqrt{\frac{3}{4}}$~~
- No radical sign in denominator of fraction.
 ~~$\frac{3}{\sqrt{5}}$~~

$$\sqrt{16} = \sqrt{4 \cdot 4} = \sqrt{4} \cdot \sqrt{4} = 2 \cdot 2 = 4$$

$$\sqrt{2} \cdot \sqrt{8} = \sqrt{2 \cdot 8} = \sqrt{16} = 4$$

Product Property of Radicals

Words The square root of a product equals the product of the square roots of the factors.

Algebra $\sqrt{ab} = \sqrt{a} \cdot \sqrt{b}$
 where $a \geq 0$ and $b \geq 0$

Example $\sqrt{4x} = \sqrt{4} \cdot \sqrt{x} = 2\sqrt{x}$

$$\sqrt{32} = \sqrt{16 \cdot 2} = \sqrt{16} \cdot \sqrt{2} = 4\sqrt{2}$$

$$2\sqrt{8} = 2\sqrt{4 \cdot 2} = 2\sqrt{4} \cdot \sqrt{2} = 2 \cdot 2 \cdot \sqrt{2} = 4\sqrt{2}$$

$$\sqrt{9x^3} = \sqrt{9 \cdot x^2 \cdot x} = \sqrt{9} \cdot \sqrt{x^2} \cdot \sqrt{x} = 3x\sqrt{x}$$

$$\sqrt{3x} \cdot 4\sqrt{x} = 4\sqrt{3x^2} = 4x\sqrt{3}$$

$$\sqrt{7xy^2} \cdot 3\sqrt{x} = 3\sqrt{7x^2y^2} = 3xy\sqrt{7}$$

Quotient Property of Radicals

Words The square root of a quotient equals the quotient of the square roots of the numerator and denominator.

Algebra $\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}$ where $a \geq 0$ and $b > 0$

Example $\sqrt{\frac{16}{25}} = \frac{\sqrt{16}}{\sqrt{25}} = \frac{4}{5}$

a. $\sqrt{\frac{13}{100}} = \frac{\sqrt{13}}{\sqrt{100}}$
 $\frac{\sqrt{13}}{10}$

b. $\sqrt{\frac{7}{x^2}} = \frac{\sqrt{7}}{\sqrt{x^2}}$
 $\frac{\sqrt{7}}{x}$

3. $\sqrt{20}$

$\sqrt{4 \cdot 5} = \sqrt{4} \cdot \sqrt{5}$
 $2\sqrt{5}$
 $\sqrt{16 \cdot 3}$
 $4\sqrt{3}$

4. $\sqrt{48}$

$\sqrt{4 \cdot 12}$
 $2\sqrt{12} = 2\sqrt{4 \cdot 3}$
 $2 \cdot 2\sqrt{3}$
 $4\sqrt{3}$

15. $2\sqrt{a^4b^5}$

16. $\sqrt{64s^4t^3}$

19. $\sqrt{\frac{4}{49}}$

20. $\sqrt{\frac{7}{81}}$

Rationalize the Denominator

$$\frac{5}{\sqrt{7}} \cdot \frac{\sqrt{7}}{\sqrt{7}} = \frac{5\sqrt{7}}{7}$$

$$\frac{\sqrt{2}}{\sqrt{3b}} \cdot \frac{\sqrt{3b}}{\sqrt{3b}} = \frac{\sqrt{6b}}{3b}$$

1. $\frac{1}{\sqrt{2x}}$

$$\cdot \frac{\sqrt{2x}}{\sqrt{2x}}$$

$$\frac{\sqrt{2x}}{2x}$$

32. $\sqrt{\frac{2x^2}{5}}$

$$\frac{\sqrt{2x^2}}{\sqrt{5}} \cdot \frac{\sqrt{5}}{\sqrt{5}}$$

$$\frac{\sqrt{10x^2}}{5} = \frac{x\sqrt{10}}{5}$$

33. $\sqrt{\frac{8}{3n^3}} = \frac{\sqrt{8}}{\sqrt{3n^3}} \cdot \frac{\sqrt{3n}}{\sqrt{3n}}$

$$\frac{\sqrt{24n^3}}{3n^3} = \frac{\sqrt{4 \cdot 6 \cdot n^2 \cdot n}}{3n^3}$$

$$\frac{2\sqrt{6n}}{3n^3} = \frac{2\sqrt{6n}}{3n^2}$$

Add and Subtract Radicals

Can only combine like radicals (like terms)

$$\begin{aligned} & 4\sqrt{10} + \sqrt{13} - 9\sqrt{10} = \\ & -5\sqrt{10} + \sqrt{13} \end{aligned}$$

$$5\sqrt{3} + \sqrt{48} = 4\sqrt{16 \cdot 3}$$

$$5\sqrt{3} + 4\sqrt{3} = 9\sqrt{3}$$

~~$$0 = 6\sqrt{3}$$~~

$$3 \cdot 5\sqrt{9 \cdot 6}$$

34. $2\sqrt{2} + 6\sqrt{2}$

$$8\sqrt{2}$$

$$\begin{array}{r} 1x - 6x \\ -2x \end{array}$$

35. $\sqrt{5} - 6\sqrt{5}$

$$-5\sqrt{5}$$

36. $2\sqrt{6} - 5\sqrt{54}$

$$\begin{aligned} & 2\sqrt{6} - 15\sqrt{6} \\ & -13\sqrt{6} \end{aligned}$$

Multiply Radical Expressions

$$\sqrt{5} \cdot \sqrt{4}$$

a. $\sqrt{5}(4 - \sqrt{20}) =$

$$4\sqrt{5} - \sqrt{100}$$

$$4\sqrt{5} - 10$$

41. $\sqrt{6}(7\sqrt{3} + 6)$

$$7\sqrt{18} + 6\sqrt{6}$$

$$21\sqrt{2} + 6\sqrt{6}$$

b. $(\sqrt{7} + \sqrt{2})(\sqrt{7} - 3\sqrt{2})$

$$7\sqrt{18} - 3\sqrt{7 \cdot 2}$$

42. $\sqrt{3}(6\sqrt{2} - 4\sqrt{3})$