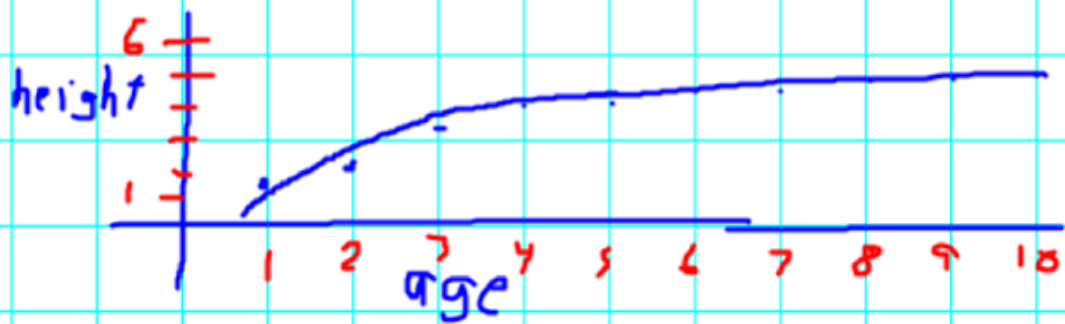


3.2 Logarithmic Functions

Logs are the inverse of exponents

- the graphs start fast and level out.



Definition of Log Functions

for $x > 0$, $b > 0$, $b \neq 1$

$$y = \log_b X$$

↑
exponent

↑
base

$$y \text{ iff } b^y = X$$

log solves
for the exponent

ex. $3 = \log_2 8$ is equivalent to $2^3 = 8$

base^{exponent} = power
and $\log_{\text{base}} \text{Power} = \text{expon}$

Rewrite

$$1) \log_4 64 = 3$$

$4^3 = 64$

$$2) \log_{32} 4 = \frac{2}{5}$$

$32^{\frac{2}{5}} = 4$

$$3) 5^3 = 125$$

$\log_5 125 = 3$

$$4) 6^{-2} = \frac{1}{36}$$

$\log_6 \frac{1}{36} = -2$

Evaluate logs

$$f(x) = \log_2 x, \quad x = 32$$

$$f(32) = \log_2 32$$

$$2^y = 32$$

$$y = 5$$

$$y = \log_4 x$$

$$x = 16$$

$$4^y = 16$$

$$y = 2$$

Common Logs

LOG

base 10

$\log_{10} 100$ same as $\log 100$

calc

$$\log 10 = 1$$

$$\log 145 \approx 2.16$$

$$\log 100 = 2$$

$$\log 2.5 \approx .396$$

$$\log 1000 = 3$$

$$\log(-10) = \text{error}$$

Natural Log

LN

$\log_e X$ same as $\ln X$

calc

$$\ln 2 \approx .693$$

$$\ln 0.3 \approx -1.203$$

$$\ln e^2 = 2 \quad \ln e^5 = 5$$

$$8^x = 352$$

$$\ln 8^x = \ln 352$$

$$\frac{x \ln 8}{\ln 8} = \frac{\ln 352}{\ln 8} \quad 8^{2.819} \approx 352$$

$$x = \frac{\ln 352}{\ln 8} \approx 2.819$$

Properties of Common and Natural Logarithms

$$\log_b 1 = 0$$

$$\ln 1 = 0 \quad e^0 = 1$$

$$\log_b b = 1$$

$$\ln e = 1 \quad e^1 = e$$

$$\log_b b^x = x \quad \& \quad b^{\log_b x} = x \quad \ln e^x = x \quad \& \quad e^{\ln x} = x$$

$$\log_b x = \log_b y \text{ then } x = y \quad \ln x = \ln y \text{ then } x = y$$