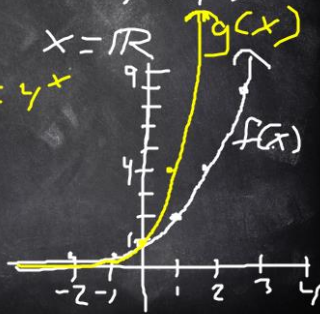


3.1 Exponential Functions and their Graphs

Def. $f(x) = a^x$ $a > 0, a \neq 1$

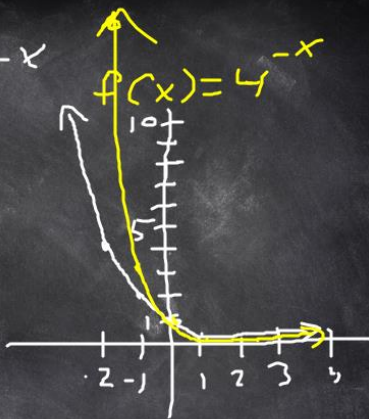
Graphing $f(x) = 2^x$ $g(x) = 4^x$

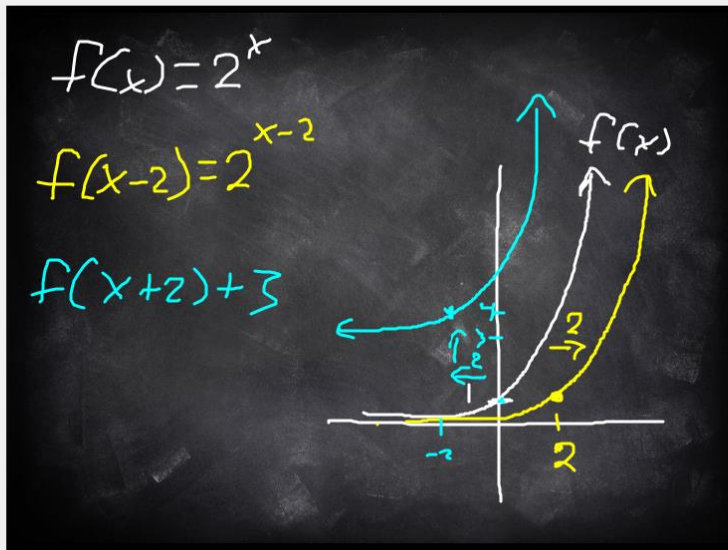
| | | | | | |
|------|------|-----|---|---|----|
| x | -2 | -1 | 0 | 1 | 2 |
| f(x) | 1/4 | 1/2 | 1 | 2 | 4 |
| g(x) | 1/16 | 1/4 | 1 | 4 | 16 |



ex graph $f(x) = 2^{-x}$

| | | |
|----|------|------|
| x | f(x) | g(x) |
| -2 | 4 | 16 |
| -1 | 2 | 4 |
| 0 | 1 | 1 |
| 1 | 1/2 | 1/4 |
| 2 | 1/4 | 1/16 |





Natural Base e & Compound Interest
Compound Interest Formulas

- For 'n' compounding per year
 $A = P\left(1 + \frac{r}{n}\right)^{nt}$
- For continuous compounding
 $A = Pe^{rt}$



Save

ex
Compound Interest

P = Principle rate 3%
ex r = .03

| Time (yrs) | Balance |
|------------|--------------------------------------------------------------------------------------------------------------------------|
| 0 | $P = P$ $1000 = 1000$ |
| 1 | $P_1 = P(1+r)$ $1000(1+.03) = 1030$ |
| 2 | $P_2 = P_1(1+r)$ or $= P(1+r)(1+r)$ or $= P(1+r)^2$ $= 1060.90$ |
| 3 | $P_3 = P(1+r)^3$ |
| ... | |
| t | $P_t = P(1+r)^t$ |



Save

Compounding quarterly, monthly

$$A = P \left(1 + \frac{r}{n}\right)^{nt}$$

A = total amt.
P = initial principle
r = int. rate
n = # of comp. per year
t = # of years





Save

$$A = P \left(1 + \frac{r}{n}\right)^{nt}$$

Money for ten years
compounded...

$P=1000$ $r=3\%$ $t=10$ years

quarterly monthly daily

$$A = 1000 \left(1 + \frac{.03}{4}\right)^{4 \cdot 10} = 1000 \left(1 + \frac{.03}{12}\right)^{12 \cdot 10} = 1000 \left(1 + \frac{.03}{365}\right)^{365 \cdot 10}$$

$$A = 1348.35 = \$1349.35 = \$1349.84$$

every hour? every minute? seconds?



Save

Continuous Compounding

Through Algebra Magic

$$A = P \left[\left(1 + \frac{r}{x}\right)^x \right]^{rt}$$

explore $\left(1 + \frac{1}{x}\right)^x$ for
increasing values of x

$f(x) = \left(1 + \frac{1}{x}\right)^x$

$f(10) = 2.5937$
 $f(100) = 2.7048$
 $f(1000) = 2.7169$
 $f(10000) = 2.7183$



$e = 2.718281828\dots$

↑
 Natural base
 continuous compounding
 $A = Pe^{rt}$
 $P = 12000$ interest 4% $t = 5$ yr
 $A = 12000e^{(0.04 \cdot 5)}$
 $= \$14,656.83$

Natural base 'e'
 $e \approx 2.718\dots$

Calc
 $\frac{e^x}{\ln}$

↑

Population growth
 Fruit fly population increases exponentially according to the model $Q(t) = 20e^{0.03t}$
 $t = \text{time (hrs)}$ where $t \geq 0$

a. initial population?
 $Q(0) = 20e^{0.03(0)}$
 $= 20e^0$
 $= 20(1)$
 $= 20$

b. 72 hrs
 $Q(72) = 20e^{0.03(72)}$
 $Q(72) = 173$