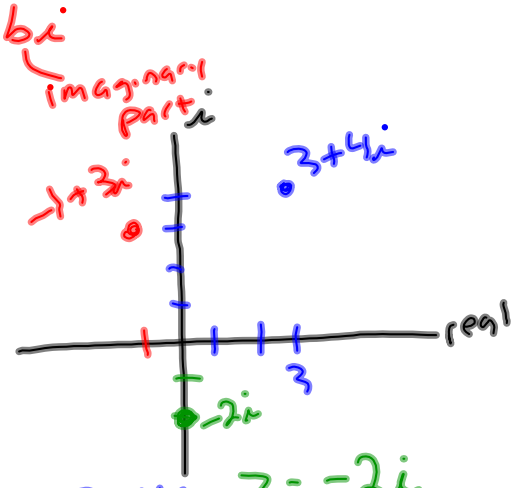
Jan. 14 # Complex Number in Polar Form

$a + bi$ ← real part, imaginary part

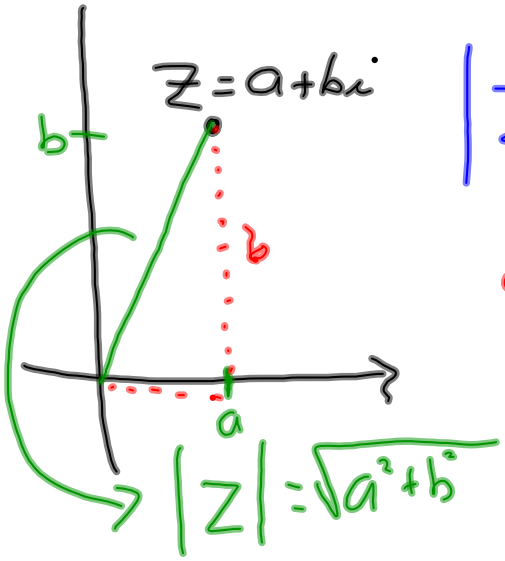


$z = 3+4i$ $z = -2i$

$z = -\frac{1}{2} + \frac{3}{4}i$

Absolute Value of Complex Numbers

→ distance from zero



$|z| = |a+bi| = \sqrt{a^2+b^2}$

ex $z = 3+4i$

$|z| = \sqrt{3^2+4^2}$

$\sqrt{9+16}$

$|z| = 5$

Practice

Find absolute value of Z when...

$Z = 5 + 12i$ $Z = 2 - 3i$ $Z = -4i$

$|Z| = 13$

$|z| = \sqrt{2^2 + (-3)^2}$

$= \sqrt{4 + 9}$

$|z| = \sqrt{13}$

$|z| = \sqrt{0^2 + (-4)^2}$

$= \sqrt{0 + 16}$

$|z| = 4$

Do now

Find absolute value of Z

① $Z = 6 + 8i$ ② $Z = -3 + 5i$ ③ $z = 6 - i$

$|z| = \sqrt{a^2 + b^2}$

$\sqrt{6^2 + 8^2}$

$\sqrt{36 + 64}$

$|z| = 10$: r

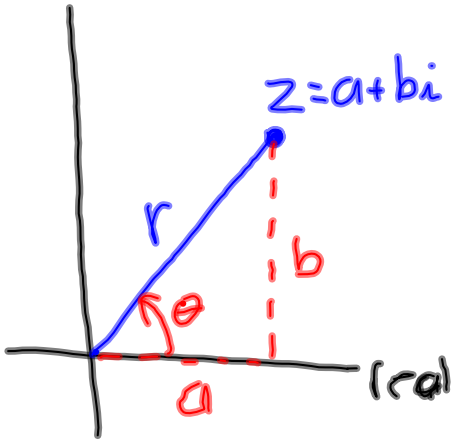
$|z| = \sqrt{(-3)^2 + 5^2}$

$\sqrt{9 + 25}$

$\sqrt{34}$

$\sqrt{37}$

1/15 Polar Form of a Complex Number



$z = a + bi$ is rectangular form

since $\cos\theta = \frac{a}{r}$ & $\sin\theta = \frac{b}{r}$

$a = r\cos\theta \quad b = r\sin\theta$

Therefor

$z = r\cos\theta + r\sin\theta i$

$z = r(\cos\theta + i\sin\theta)$ Polar form

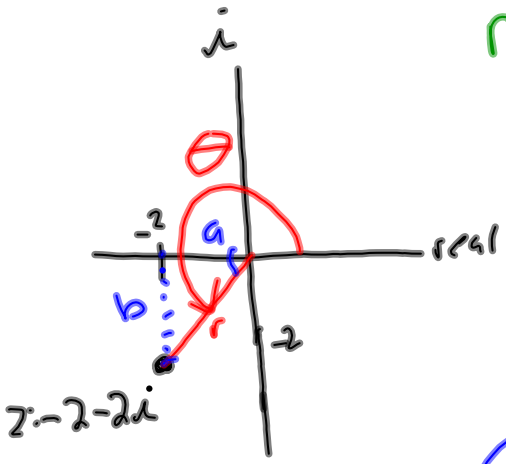
$z = r\,\mathrm{cis}\,\theta$ shorthand

$r\,\mathrm{cis}\,\theta = r(\cos\theta + i\sin\theta)$

cos — imaginary unit — sin

note: $2\,\mathrm{cis}\,30^\circ = 2(\cos 30^\circ + i\sin 30^\circ)$

Write $-2-2i$ in polar form



$r = |z| = \sqrt{(-2)^2 + (-2)^2}$

$= \sqrt{4+4} = \sqrt{8}$

$r = 2\sqrt{2}$

$\theta = \tan^{-1}\frac{b}{a}$

$\tan^{-1}\left(\frac{-2}{-2}\right) = \tan^{-1}(1) = 45^\circ$

$\theta = 225^\circ$

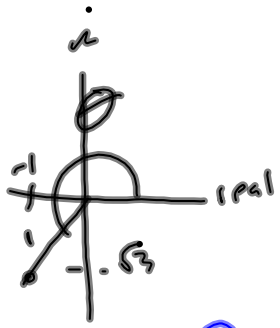
$2\sqrt{2}\,\text{cis}\,225^\circ$

Practice:

Write $z = -1 - i\sqrt{3}$ in polar form

$a = -1 \quad b = -\sqrt{3}$

note: $-i\sqrt{3}$ not $-\sqrt{3}i$ to avoid confusion



$r = |z| = \sqrt{(-1)^2 + (-\sqrt{3})^2}$

$= \sqrt{4}$

$r = 2$

$\theta = \tan^{-1}\frac{-\sqrt{3}}{-1} = \tan^{-1}\sqrt{3}$

$\theta = 60^\circ$

$+180$

$2\,\text{cis}\,240^\circ$

Convert Complex Number from Polar to Rectangular

$z = 2\text{cis}\,60^\circ \rightarrow$ rectangular

1st rewrite in long form: $= 2(\cos 60^\circ + i \sin 60^\circ)$

2nd find trig. values: $= 2\left(\frac{1}{2} + i\frac{\sqrt{3}}{2}\right)$

3rd distribute: $= 1 + i\sqrt{3}$

Done $z = \underbrace{1}_{a} + \underbrace{i\sqrt{3}}_{b}$